DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

TEST BOOKLET NO-25015

MAIN PAPER

Time : 2:00 hrs

Total Marks: 100

MATHEMATICS

INSTRUCTIONS TO CANDIDATES

Read the instructions carefully before answering the questions: -

1. This Test Booklet consists of 12 (twelve) pages and has 50 (fifty) **items (questions)**.
2. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
3. **Please note that it is the candidate's responsibility to fill in the Roll Number and other required details carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the OMR Answer Sheet liable for rejection.**
4. Do not write anything else on the OMR Answer Sheet except the required information. Before you proceed to mark in the OMR Answer Sheet, please ensure that you have filled in the required particulars as per given instructions.
5. Use **only Black Ball Point Pen** to fill the OMR Answer Sheet.
6. **This booklet consists of Multiple Choice-based Questions.** The answers to these questions have to be marked in the **OMR Answer Sheet** provided to you.
7. Each item (question) comprises of 04 (four) responses (answers). You are required to select the response which you want to mark on the OMR Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
8. After you have completed filling in all your responses on the OMR Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***the OMR Answer Sheet*** .You are permitted to take the Test Booklet with you.
9. **Penalty for wrong answers in Multiple Choice-based Questions:**
   THERE WIL BE **PENALTY** FOR WRONG ANSWERS MARKED BY A CANDIDATE.
   (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to the question will be deducted as penalty.
   (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to the question.
   (iii) If a question is left blank. i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

MULTIPLE CHOICE BASED QUESTIONS

Instructions for Questions 1 to 50:

- Choose the correct answer for the following questions
- Each question carries 2 marks (50 x 2 = 100 marks)

Q1 Let S={(-1,1,0),(1,-1,0)} be a subset of vector space $\mathbb{R}^3(\mathbb{R})$. The span of S is a line

(a) parallel to x-axis
(b) perpendicular to x-axis
(c) parallel to z-axis
(d) perpendicular to z-axis

Q2 The value of line integral $\int_C (xdy - ydx)$,where C is the unit circle centered at origin (0,0), is

(a) 2π
(b) π
(c) $\pi/2$
(d) None of these

Q3 The dimension of space of solutions to the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ over the field of complex numbers is

(a) 1
(b) 0
(c) 3
(d) 2

Q4 The nullity of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$defined by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$is

(a) 0
(b) 1
(c) 3
(d) 2

Q5 The minimal polynomial for the identity operator on a finite dimensional vector space is

(a) x
(b) x+1
(c) x-1
(d) x^2+1

Q6 An eigen value of the transformation $T: \mathbb{C}^3 \to \mathbb{C}^3$ defined by $T(x,y,z) = (ix, -iy, z)$ is

(a) 0
(b) i+1
(c) i-1
(d) -i

Q7 The equation of xy-plane in $\mathbb{R}^3$ is

(a) z=0
(b) y=0
(c) x=0
(d) None of them

Q8 Let x and y be two perpendicular vectors in an inner product space. Then $\|x+y\|^2$ equals to

(a) $\|x\|^2$-$\|y\|^2$
(b) $\|x\|^2$+$\|y\|^2$
(c) 2($\|x\|^2$+$\|y\|^2$)
(d) None of them

Q9 Let A be an orthogonal set of non-zero vectors of an inner product space V. Then

(a) A is subspace of V
(b) A is linearly dependent subset of V
(c) A is linearly independent subset of V
(d) None of them

Q10 Which of the following represent a quadratic form of order two ?

(a) x^2-xy+y^2
(b) x+y
(c) $2x^3$+$2y^3$
(d) None of these

Q11 Which of the following plane is perpendicular to x-axis in $\mathbb{R}^3$?

(a) z = 0
(b) y = 0
(c) x = 0
(d) None of these

Q12 The value of k for which x^2-$2y^2$-kxy^2 represent a quadratic form, is

(a) 3
(b) 2
(c) 1
(d) 0

Q13 The equation of diameter of the circle $x^2+y^2-6x+2y-8=0$ which passes through the origin is

(a) $x+3y=0$
(b) $3x+y=0$
(c) $x+y=0$
(d) $x-y=0$

Q14 Which of the following statements is TRUE?

(a) Q is both a subring and ideal of R
(b) Q is a subring but not an ideal of R
(c) Q is neither a subring nor an ideal of R
(d) Q is not a subring but an ideal of R

Q15 The equation of theord of the circle $x^2+y^2=9$ whose mid point is (1,-2), is

(a) $x-2y=5$
(b) $x-y=3$
(c) $2x-y=4$
(d) None of these

Q16 In Linear Programming Problem (L.P.P.), feasible region is always

(a) a convex set
(b) a concave set
(c) a bounded concave set
(d) None of these

Q17 The equation of line perpendicular to the plane $x+y+z=1$ is

(a) $x=2y=z$
(b) $x=y=2z$
(c) $2x=y=2z$
(d) $x=y=z$

Q18 The equation of plane passing through (1, 2, 3) and perpendicular to the line $x = \frac{y}{2} = \frac{z}{3}$ is

(a) $x+2y+3z=14$
(b) $2x+y+3z=13$
(c) $x+3y+2z=13$
(d) $3x+2y+z=12$

Q19 Let X be a compact metric space. Which of the following statements is true?

(a) All subsets of X are compact
(b) No subset of X is compact
(c) Closed subsets of X are compact
(d) Open subsets of X are compact

Q20 A coin is tossed until a tail appears. The expectation of the number of tosses required is

(a) 2
(b) 3
(c) 1
(d) 0

Q21 Which of the following statements is true?

(a) An algebraic extension is a finite extension.
(b) A finitely generated extension is a finite extension.
(c) A finite extension is algebraic extension.
(d) A simple extension is finite extension.

Q22 An unbiased coin is tossed six times. The probability of getting at most 3 tails is

(a) $\frac{21}{64}$

(b) $\frac{15}{64}$

(c) $\frac{21}{32}$

(d) $\frac{15}{32}$

Q23 Which of the following subset of real line is compact

(a) (c, d)

(b) $[c, d)$
(c) $(c, d]$
(d) $[c, d]$

Q24 The area of triangle bounded by lines $y=x$, $y=2x$ and $x=2$ is

(a) 1
(b) 2
(c) 3
(d) 4

Q25 A relation $\sim$ on $\mathbb{Z}$is defined by $a \sim b$ if $a + b$ is even. The number of equivalence classes of the relation $\sim$ is

(a) 4
(b) 1
(c) 3
(d) 2

Q26 Let $f(x) = x^3 + 5x$ be a polynomial over $\mathbb{Z}_6$. The number of zeros of $f(x)$ in $\mathbb{Z}_6$ are

(a) 3
(b) 4
(c) 5
(d) 6

Q27 Let a and b be two elements of a group G such that $ab=ba$ and $(o(a), o(b))=1$, where $o(a)$ denotes the order of element a in G. Then

(a) $o(ab) = o(a) + o(b)$
(b) $o(ab) = o(a) - o(b)$
(c) $o(ab) = o(a)o(b)$
(d) None of these

Q28 The additive group of rational numbers is

(a) not abelian
(b) not cyclic
(c) not infinite
(d) None of these

Q29 Let G be a group of order 35. The number of distinct subgroups of order 7 are

(a) 1
(b) 2
(c) 3
(d) None of these

Q30 The quotient group $\mathbb{R}/\mathbb{Z}$is isomorphic to

(a) the circle group
(b) the additive group of rationals
(c) the additive group of complex numbers
(d) None of these

Q31 If an element a of a group G satisfies $a^2=a$, then the element a is

(a) the identity of the group
(b) an element of order 2
(c) an element of order 4
(d) None of these

Q32 Which of the following rings is a field ?

(a) $2\mathbb{Z}$
(b) $\mathbb{Z}_5$
(c) $\mathbb{Z}_8$
(d) $\mathbb{Q}x\mathbb{Q}$

Q33 The quotient group $3\,\mathbb{Z}/12\,\mathbb{Z}$ is isomorphic to

(a) $\mathbb{Z}_4$
(b) $\mathbb{Z}_{15}$
(c) $\mathbb{Z}_9$
(d) None of these

Q 34 Let $f(x) = x^2 + x + 1$be a polynomial over $\mathbb{Z}_3$. The polynomial $f(x)$

(a) has two distinct roots in $\mathbb{Z}_3$
(b) is irreducible over $\mathbb{Z}_3$
(c) has only one root in $\mathbb{Z}_3$with multiplicity 2
(d) None of these

Q35 If $\vec{f} = x^2z\mathbf{i} - 2y^3z^2\mathbf{j} + xy^2\mathbf{k}$, then $div\vec{f}$is

(a) -1
(b) 3
(c) -2
(d) -3

Q36 Which of the following functions is one-to-one ?

(a) $f: \mathbb{R} \to \mathbb{R}$defined by $f(x) = |x|$
(b) $f: \mathbb{R} \to \mathbb{R}$defined by $f(x) = 3x - 1$
(c) $f: \mathbb{R} \to \mathbb{R}$defined by $f(x) = x^2$
(d) None of these

Q37 The left hand limit of $lim\, x - [x]$, when $x \to 3$ is

(a) 0
(b) 2
(c) 3
(d) 1

Q38 Let$\vec{a}.\vec{b} = \vec{a}.\vec{c}and\vec{a}x\vec{b} = \vec{a}x\vec{c}$,where$\vec{a}$ is a non-zero vector. Which of the following statements is incorrect ?

(a) $\vec{a}$is perpendicular to$\vec{b} - \vec{c}$
(b) $\vec{a}$is parallel to$\vec{b} - \vec{c}$

(c) $\vec{b} \neq \vec{c}$
(d) None of these

Q39 The value of double integral $\int_D (x+y)\,dxdy$,where D is the region bounded by $0 \leqslant x \leqslant 1$and $1 \leqslant y \leqslant e^x$,is

(a) $\frac{e^2-3}{4}$

(b) $\frac{e^2-4}{3}$

(c) $\frac{e^2+3}{4}$

(d) $\frac{e^2+4}{3}$

Q40 The dimension of subspace W={(x,y,z)|x+y+z=0} of vector space $\mathbb{R}^3(\mathbb{R})$ is

(a) 1
(b) 2
(c) 3
(d) 0

Q41 Let $f(x) = Ax^4, 0 \leqslant x \leqslant 1$,be a probability density function of a continuous random variable X. Then the value of A *is*

(a) $\frac{1}{4}$
(b) 5
(c) 4
(d) $\frac{1}{5}$

Q42 The expectation of getting a prime number on throwing an unbiased dice is

(a) $\frac{5}{3}$

(b) $\frac{2}{7}$

(c) $\frac{7}{2}$

(d) $\frac{3}{5}$

Q43 Let X be a discrete random variable, then the variance $V(aX+b)$ is equal to

(a) $aV(X)$
(b) $a^2V(X)$
(c) $a^2V(X)+b^2$
(d) $aV(X)+b$

Q44 An algebraically closed field is

(a) Finite and perfect.
(b) Infinite and perfect.
(c) Finite but not perfect.
(d) Infinite but not perfect

Q45 The mean and standard deviation of a binomial distribution X are 4 and $\frac{2}{\sqrt{3}}$, respectively. The value of P[X=0]

(a) $\frac{1}{729}$
(b) $\frac{728}{729}$
(c) $\frac{5}{729}$
(d) $\frac{724}{729}$

Q46 Let X be a Poisson variate such that $P[X = 1] = 2P[X = 2]$. The variance of X is

(a) 4
(b) 3
(c) 1
(d) 2

Q47 The number of units in the rings $\mathbb{Z}_5$ and $\mathbb{Z}_6$ are

(a) 2 and 5
(b) 5 and 2
(c) 1 and 2
(d) 2 and 1

Q48 If $\vec{r}$= x**i** + y**j** + z**k** and $\vec{a}$is a fixed vector, then $grad(\vec{r}.\vec{a})$is

(a) $\vec{a}$
(b) $\vec{r}$
(c) $\vec{a}.\vec{r}$
(d) $\vec{a}x\vec{r}$

Q49 If$\vec{r}x\vec{a} = \vec{b}x\vec{a}$,then the value of $\vec{r}$is

(a) $\vec{a} + t\vec{b}$
(b) $\vec{b} + t\vec{a}$
(c) $\vec{a} + \vec{b}$
(d) $\vec{a} - \vec{b}$

Q50 The value of line integral $\int_C (x^2 + y^2)\, dy$, where C is the curve given by $x(t)=at^2$, $y(t)=2at$, $0 \leqslant t \leqslant 1$,

(a) $\frac{46}{15}a^3$

(b) $\frac{45}{16}a^3$

(c) $\frac{45}{16}a^2$

(d) $\frac{46}{15}a^2$

.....X. .....